AFM Test Review   Name _____

Rational Functions, Variations, Rational Equations and Inequalities

1. Find the domain, the vertical asymptote(s), hole(s), horizontal asymptote(s), and oblique asymptote(s) for each of the following.

a. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 4x + 3}$ b. $g(x) = \dfrac{2x^2 - 5x + 2}{x^2 - 4}$ c. $h(x) = \dfrac{x^3 - 2x^2 + x}{x^2 - 9}$ d. $f(x) = \dfrac{x^2 + 2x - 3}{x^3 - 4x}$

VA: $X = 3, 1$
Hole: None
HA: $y = 1$
OA: None
Domain: $\mathbb{R} \notin 3, 1$

VA: $X = -2$
Hole: $X = 2$
HA: $y = 2$
OA: None
Domain: $\mathbb{R} \notin -2, 2$

VA: $X = -3, 3$
Hole: None
HA: None
OA: $y = X - 2$
Domain: $\mathbb{R} \notin -3, 3$

VA: $X = 0, 2, -2$
Hole: None
HA: $y = 0$
OA: None
Domain: $\mathbb{R} \notin 0, 2, -2$

2. The number of days (d) needed to assemble some machines varies directly as the number of machines (m) and inversely as the number of people (p) working. If it takes 4 people 32 days to assemble 16 machines, how many days will it take 8 people to assemble 24 machines?

$= K$

$d = \dfrac{Km}{P}$ $32 = \dfrac{16K}{4}$ $K = 8$ $d = \dfrac{8(24)}{8}$

$128 = 16K$ $\boxed{d = 24 \text{ days}}$

3. The power (P), in watts, dissipated as heat in a resistor varies directly as the square of voltage (V), in volts, and inversely as the resistance (R) in ohms. If 20 volts are placed across a 20 ohm resistor, it will dissipate 20 watts. What voltage across a 10 ohm resistor will dissipate 40 watts?

$P = \dfrac{KV^2}{R}$ $20 = \dfrac{(20)^2 K}{20}$ $40 = \dfrac{V^2}{10}$ $\boxed{20 \text{ volts}}$

$400 = 400K$ $400 = V^2$

$K = 1$ $V = 20$

4. The stress (S) in the material of a pipe subject to internal pressure varies jointly with the internal pressure (P) and the internal diameter (D) of the pipe and inversely with the thickness (T) of the pipe. The stress is 100 pounds per square inch when the diameter is 5 inches, the thickness is .75 inch, and the internal pressure is 25 pounds per square inch. Find the stress when the internal pressure is 40 pounds per square inch if the diameter is 8 inches and the thickness is .50 inch.

$S = \dfrac{KPD}{T}$ $100 = \dfrac{25(5)K}{0.75}$ $S = \dfrac{(0.6)(40)(8)}{0.5}$

$75 = 125K$

$K = 0.6$ $\boxed{S = 384 \text{ lbs}/\text{in}^2}$

Solve for x. Show work.

5. $\dfrac{2+x}{3} + \dfrac{x+7}{2} = 4x+1$

6. $\dfrac{x+2}{x+3} + \dfrac{1}{x^2+2x-3} = 1$

7. $\dfrac{9}{x+5} = \dfrac{3}{x-3}$

$x = 1$

$x = 2$

$x = 7$

8. $\dfrac{3x}{x^2+x} - \dfrac{2x}{x^2+5x} = \dfrac{x+2}{x^2+6x+5}$

9. $\dfrac{10}{x^2-1} + \dfrac{2x-5}{x-1} = \dfrac{2x+5}{x+1}$

No Solution

$x = 5/3$

Solve each rational inequality. Write each solution in interval notation and graph the solution on the
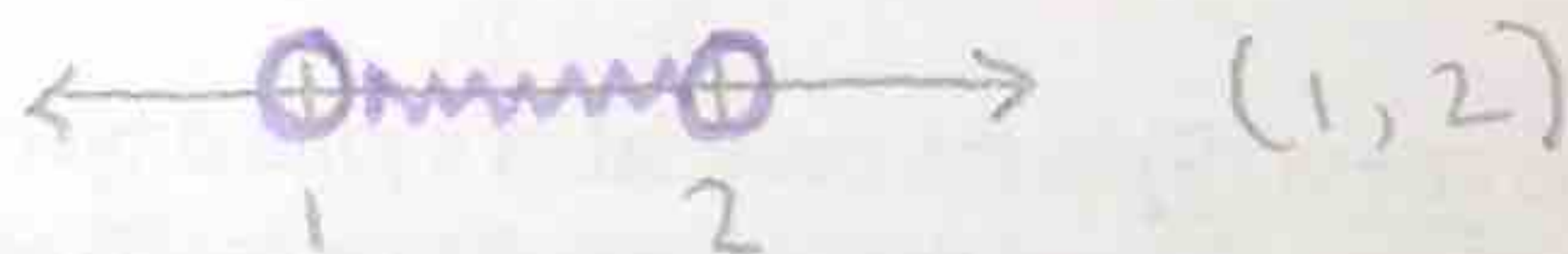number line.

10. $\dfrac{x+3}{x-2} < 0$

$x = -3$
$x = 2$.

$(-3, 2)$

11. $\dfrac{7}{x-1} > 7$ work on other
sheet

$x = 2$
$x = 1$

$(1, 2)$

12. $\dfrac{x^2-16}{x^2-4x-5} \geq 0$

$\dfrac{(x-4)(x+4)}{(x-5)(x+1)} \geq 0$

$x = 4$
$x = -4$
$x = 5$
$x = -1$
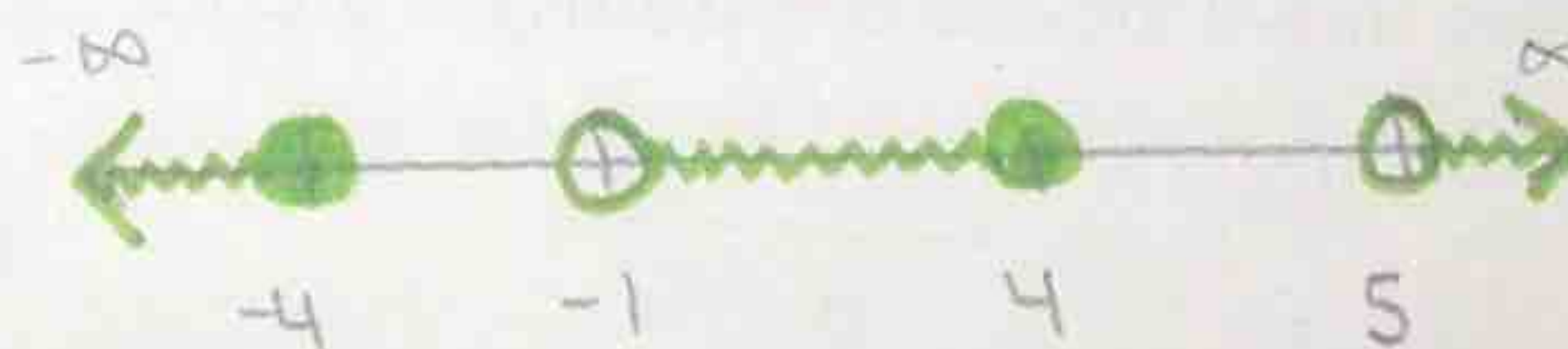
$(-\infty, -4] \cup (-1, 4] \cup (5, \infty)$

Solve. Show work.

13. The sum of a number and its reciprocal is $\frac{10}{3}$. Find the number.

$3x-1=0 \qquad x-3=0$

$3x=1 \qquad \boxed{x=3}$

$\boxed{x=1/3}$

$(3)(x) \; x + \frac{1(3)(x)}{x} = \frac{10}{3}(3)(x)$

$3x^2 + 3 = 10x$

$3x^2 - 10x + 3 = 0$

$-9 \diagdown -1$

-10

$(3x^2 - 9x)(-1x+3)$

$3x(x-3) -1(x-3)$

14. One pipe can fill a tank in 4 hours and another pipe can fill the tank in 6 hours. How long will it take to fill the tank if both pipes are used?

$\frac{x}{4} + \frac{x}{6} = 1$

$10x = 24$

$6x + 4x = 24$

$\boxed{x = 2.4 \text{ hours}}$

15. Carlos can mow a lawn in 45 minutes and Felipe can mow the same lawn in 30 minutes. How long would it take the two of them working together to mow the lawn?

$(45)(30) \; \frac{x}{45} + \frac{(45)(30)\;x}{30} = 1(45)(30)$

$30x + 45x = 1350$

$75x = 1350$

$x = 18$

$\boxed{18 \text{ minutes}}$

16. Sally and Tom working together can paint the exterior of a house in 9 days. Tom by himself can complete the job in 7 days less than Sally. How long will it take for Sally to complete the job herself? (Quadratic formula is needed)

$s(s-7) \quad \frac{9}{s} + \frac{s(s-7)\;9}{s-7} = 1 \; s(s-7)$

$9(s-7) + 9(s) = s(s-7)$

$9s - 63 + 9s = s^2 - 7s$

$18s - 63 = s^2 - 7s$

$0 = s^2 - 25s + 63$

$\boxed{2.85 \text{ days}}$

$s = \frac{25 \pm \sqrt{(25)^2 - 4(1)(63)}}{2}$

$s = \frac{25 \pm 19.3}{2}$

$\cancel{22.15}$

$\boxed{2.85}$

17. A boat travels 246 miles downstream in the same time it takes to travel 180 miles upstream. The speed of the current in the stream is 5.5 mph. Find the speed of the boat in still water.

same means equal!

$x =$ speed of boat

$\text{Speed} = \boxed{35.5 \text{ mph}}$

on sheet!

18. To determine the number of fish in a lake, a conservationist catches 225 fish, tags them, and throws them back into the lake. Later, 108 fish are caught and 15 of them are found to be tagged. Estimate how many fish are in the lake.

$\frac{\text{Total # of trout}}{\text{# tagged}} = \frac{\text{Total # of trout}}{\text{# tagged}}$

$\frac{x}{225} = \frac{108}{15}$

$15x = 24300$

$\boxed{x = 1620 \text{ trout}}$

Rationals Test Review

1a. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 4x + 3} = \dfrac{(x-2)(x+1)}{(x-3)(x-1)}$

b. $g(x) = \dfrac{2x^2 - 5x + 2}{x^2 - 4} = \dfrac{(2x-1)(x-2)}{(x-2)(x+2)}$ ← POD!

$\times \begin{matrix} & 4 \\ -4 & & -1 \\ & -5 \end{matrix}$

$(2x^2 - 4x)(-1x + 2)$

$2x(x-2) - 1(x-2)$

$(2x-1)(x-2)$

c. $h(x) = \dfrac{x^3 - 2x^2 + x}{x^2 - 9} = \dfrac{x(x^2 - 2x + 1)}{(x-3)(x+3)} = \dfrac{x(x-1)(x-1)}{(x-3)(x+3)}$

$$x^2 + 0x - 9 \overline{)\begin{array}{l} \; x - 2 \\ x^3 - 2x^2 + 1x + 0 \\ \underline{-x^3 + 0x^2 + 9x} \\ -2x^2 + 10x + 0 \\ \underline{+2x^2 + 0x + 18} \\ R = -18 \end{array}}$$

d. $f(x) = \dfrac{x^2 + 2x - 3}{x^3 - 4x} = \dfrac{(x+3)(x-1)}{x(x^2 - 4)} = \dfrac{(x+3)(x-1)}{x(x-2)(x+2)}$

5.

$$\frac{2+x}{\cancel{6}^{(3)(2)}} + \frac{x+7}{\cancel{2}^{(3)(2)}} = (4x+1)(3)(2) \qquad LCD: (3)(2)$$

$$2(2+x) + 3(x+7) = 6(4x+1)$$
$$2x + 4 + 3x + 21 = 24x + 6$$
$$5x + 25 = 24x + 6$$
$$-19x + 25 = 6$$
$$-19x = -19$$
$$x = 1$$

6.

$$\frac{x+2}{x+3}^{(x+3)(x+-1)} + \frac{1}{(x+3)(x-1)}^{(x+3)(x-1)} = 1^{(x+3)(x-1)} \qquad LCD: (x+3)(x-1)$$

$$(x+2)(x-1) + 1 = (x+3)(x-1)$$
$$x^2 + 2x - 1x - 2 + 1 = x^2 - 1x + 3x - 3$$
$$\cancel{x^2} + 1x - 1 = \cancel{x^2} + 2x - 3$$
$$-x^2 \qquad -x^2$$

$$\begin{array}{r} 1x - 1 = 2x - 3 \\ -1x + 3 \quad -1x + 3 \\ \hline 2 = x \end{array}$$

7.

$$\frac{9}{x+5} = \frac{3}{x-3} \qquad \text{cross multiply!}$$

$$9(x-3) = 3(x+5)$$
$$9x - 27 = 3x + 15$$
$$6x = 42$$
$$x = 7$$

8.

$$\frac{3x}{\cancel{x(x+1)}}\overset{\cancel{x(x+1)(x-5)}}{} - \frac{2x}{\cancel{x(x+5)}}\overset{\cancel{x(x+1)(x-5)}}{} = \frac{x+2}{\cancel{(x+5)(x+1)}}\overset{\cancel{x(x+1)(x-5)}}{} \qquad LCD: x(x+1)(x+5)$$

$$3x(x-5) - 2x(x+1) = (x+2)(x)$$
$$3x^2 - 15x - 2x^2 - 2x = x^2 + 2x$$
$$x^2 - 17x = x^2 + 2x$$
$$-17x = 2x$$
$$+17x \qquad +17x$$
$$\overline{}$$
$$0 = 19x$$
$$0 = x \quad \leftarrow \text{restriction! can't work!}$$

9.

$$\frac{10}{\cancel{(x-1)(x+1)}}\overset{\cancel{(x+1)(x-1)}}{} + \frac{(2x-5)}{\cancel{(x-1)}}\overset{\cancel{(x+1)(x-1)}}{} = \frac{(2x+5)}{\cancel{(x+1)}}\overset{\cancel{(x+1)(x-1)}}{} \qquad LCD: (x+1)(x-1)$$

$$10 + (2x-5)(x+1) = (2x+5)(x-1)$$
$$10 + 2x^2 + 2 \ldots 5x \ldots = 2x^2 - 2x + 5x - 5$$
$$2x^2 - 3x + 5 = 2x^2 + 3x - 5$$
$$-3x \ldots = 3x - 5$$
$$-6x = -10$$
$$x = \frac{-10}{-6} = \frac{5}{3}$$

11.

$$\frac{7}{x-1} - 7 > 0 \qquad\qquad \frac{7 - 7x + 7}{x-1} > 0 \qquad\qquad -7x + 14 = 0$$

$$\frac{7}{x-1} - \frac{7(x-1)}{(x-1)} > 0 \qquad\qquad\qquad\qquad -7x = -14$$

$$\qquad\qquad\qquad\qquad\qquad\quad \frac{-7x+14}{x-1} > 0 \qquad\qquad x = 2$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \longrightarrow x = 1$$

17. $d = rt$

$t = \dfrac{d}{r}$ ← solve for t
b/c they said
it took the
"same time"

downstream: $t = \dfrac{246}{x+5.5}$

upstream: $t = \dfrac{180}{x-5.5}$

$x =$ the speed of the boat

upstream time = downstream time

$$\dfrac{246}{x+5.5} - \dfrac{180}{x-5.5}$$

$246(x-5.5) = 180(x+5.5)$
$246x - 1353 = 180x + 990$
$66x = 2343$
$x = 35.5$

18.